

Consistent Massive Colored Gravitons

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Abstract

A short review of the problems with the action for massive gravitons is presented. We show that consistency problems could be resolved by employing spontaneous symmetry breaking to give masses to gravitons. The idea is then generalized by enlarging the $SL(2, \mathbb{C})$ symmetry to $SL(2N, \mathbb{C}) \times SL(2N, \mathbb{C})$ which is broken to $SL(2, \mathbb{C})$ spontaneously through a non-linear realization. The requirement that the space-time metric is generated dynamically forces the action constructed to be a four-form. It is shown that the spectrum of this model consists of two sets of massive matrix gravitons in the adjoint representation of $SU(N)$ and thus are colored, as well as two singlets, one describing a massive graviton, the other being the familiar massless graviton.

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1 Introduction

Massive gravitons occur in higher dimensional theories of gravity when compactified to lower dimensions as an infinite tower whose masses are multiples of the Planck mass [1]. They are referred to as Kaluza-Klein (KK) gravitons. They also occur in brane models where it is possible to have an effective four-dimensional theory with ultra-light gravitons in addition to a massless graviton [2]. These are referred to as multigravity models. They are also present in noncommutative geometry for spaces which are products of a discrete space of two or more points times a manifold [3]. This can be realized as a multi-sheeted space, e.g. when $X = Y \times Z_2$, there will be a metric on every sheet, resulting in a bigravity model [4]

In the KK approach the higher dimensional metric is expanded in terms of the compactified coordinates. As an example, in five dimensions

$$g_{\mu\nu}(x, y) = \sum_{n=0}^{\infty} g_{\mu\nu n}(x) e^{iny},$$

where y is the compact coordinate in the fifth dimension. Besides the zero mode (massless graviton), there will be massive modes (massive gravitons) of masses nM_P where M_P is the Planck mass. It is not possible to obtain a graviton with a very small mass in the KK approach. This, however, is possible in brane models [5] [2] and noncommutative geometric models [4] [6].

A Lagrangian for massive spin-2 field was proposed long ago by Fierz and Pauli [7]. It is given by

$$I = -\frac{1}{4} \int d^4x \left(\partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} - 2\partial^\nu h_{\mu\nu} \partial_\lambda h^{\mu\lambda} + 2\partial^\nu h_{\mu\nu} \partial^\mu h_\lambda^\lambda - \partial_\mu h_\nu^\nu \partial^\mu h_\lambda^\lambda \right. \\ \left. + m^2 (h_{\mu\nu} h^{\mu\nu} - b h_\nu^\nu h_\lambda^\lambda) \right),$$

where, for consistency, $b = 1$ which guarantees that only five components of $h_{\mu\nu}$ propagate, instead of the expected six. The m independent part is the same as that obtained by linearizing the Einstein-Hilbert action, which is the Lagrangian for a massless spin-2 field in a Minkowski background. The

propagator for the massive graviton $h_{\mu\nu}$ is [8] [9] [10],

$$\begin{aligned}\Delta_{\mu\nu}^{\rho\sigma} = & \frac{1}{m^2 - k^2} \left(\left(\delta_\mu^\rho - \frac{k_\mu k^\rho}{m^2} \right) \left(\delta_\nu^\sigma - \frac{k_\nu k^\sigma}{m^2} \right) \right. \\ & - \frac{1}{3} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2} \right) \left(\eta^{\rho\sigma} - \frac{k^\rho k^\sigma}{m^2} \right) \\ & \left. + \frac{1-b}{2(1-b)k^2 + (1-4b)m^2} \left(\eta_{\mu\nu} + \frac{2k_\mu k_\nu}{m^2} \right) \left(\eta^{\rho\sigma} + \frac{2k^\rho k^\sigma}{m^2} \right) \right).\end{aligned}$$

When $b \neq 1$, the massive spin-2 field and the ghost of spin-0 are coupled. They only decouple for $b = 1$, the Fierz-Pauli choice. Fixing $b = 1$ could not be maintained at the quantum level. Canonical quantization shows that the modes do couple at the non-linear level [2]. The tensor $h_{\mu\nu}$ is symmetric with ten components. Unlike the massless graviton which is protected by diffeomorphism invariance, there is no gauge symmetry here and all component h_{ij} ($i, j = 1, 2, 3$) propagate, giving rise to six degrees of freedom. A massive spin-2 field has only five dynamical degrees of freedom ($2j + 1 = 5$). This implies that there is an additional component, a spin-0 ghost that does not decouple, except for the choice $b = 1$. The limit to the massless case ($m \rightarrow 0$) is singular. This is similar to the propagator of a massive spin-1 field, which is also singular in the $m \rightarrow 0$ limit. This suggests that in order to solve the problem with the singular zero mass limit, the mass of the spin-2 field should be acquired through the Higgs mechanism and spontaneous symmetry breaking. But to do this, the Lagrangian must have a gauge symmetry which should be broken. This is not possible without extending the system in such a way as not to increase the dynamical degrees of freedom. I will achieve this by employing the following idea.

Weyl formulation of a massless graviton is based on promoting the $SL(2, \mathbb{C})$ global invariance of the Dirac equation to a local one [11] [12] [13]

$$\begin{aligned}\bar{\psi} \gamma^\mu \partial_\mu \psi &\rightarrow \bar{\psi} \gamma^\mu \nabla_\mu \psi, \\ [\nabla_\mu, \nabla_\nu] &= \frac{1}{4} R_{\mu\nu}^{ab} \sigma_{ab},\end{aligned}$$

where $\nabla_\mu = \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab}$, and ω_μ^{ab} is the spin-connection. A vierbein e_μ^a is introduced to insure the gauge invariance of the gravitational action, which can be written in an index free notation

$$I_{E-H} = \frac{1}{8} \int Tr (\gamma_5 e \wedge e \wedge R),$$

where $e = e_\mu^a \gamma_a dx^\mu$, and $R = \frac{1}{2} R_{\mu\nu}^{ab} \gamma_{ab} dx^\mu \wedge dx^\nu$. The torsion defined by

$$T = de + \omega \wedge e + e \wedge \omega,$$

is set to zero, which allows for ω_μ^{ab} to be solved in terms of e_μ^a and its inverse

$$\begin{aligned} \omega_\mu^{ab}(e) &= \frac{1}{2} e^{\nu a} e^{\rho b} (\Omega_{\mu\nu\rho}(e) - \Omega_{\nu\rho\mu}(e) + \Omega_{\rho\mu\nu}(e)), \\ \Omega_{\mu\nu\rho}(e) &= (\partial_\mu e_\nu^c - \partial_\nu e_\mu^c) e_{\rho c}. \end{aligned}$$

Substituting this value of $\omega_\mu^{ab}(e)$ into the Einstein-Hilbert action gives the familiar Ricci scalar depending only on the metric $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$. The dependence on the antisymmetric part of e_μ^a cancels because of the $SL(2, \mathbb{C})$ gauge invariance of the action. The six gauge parameters of $SL(2, \mathbb{C})$ can be used to eliminate the six components in the antisymmetric part of e_μ^a . The lesson we learn from this example is that one can extend the symmetry of the system by enlarging the number of fields. In a special gauge there will be no trace of the symmetry. In this example, what protects the field $g_{\mu\nu}$ of acquiring a mass is diffeomorphism invariance of the action. It is then clear that if we have a system of two symmetric tensors, then diffeomorphism invariance can only protect one of them from becoming massive.

A coupled system of one massless graviton and one massive graviton, can be formulated as a gauge theory of $SP(4) \times SP(4)$ [14]. The group $SP(4)$ results from trading the diffeomorphism transformations of e_μ^a by a translation in internal space

$$\begin{aligned} \delta e_\mu^a &= \partial_\mu \xi^\nu e_\nu^a + \xi^\nu \partial_\nu e_\mu^a + \omega^{ab} e_{\mu b} \\ &= (\partial_\mu \xi^a + \omega_\mu^{ab}(e) \xi_b) + \omega'^{ab} e_{\mu b}, \end{aligned}$$

where the zero torsion condition is used and

$$\zeta^a = \xi^\mu e_\mu^a, \quad \omega'^{ab} = \omega^{ab} - \xi^\nu \omega_\nu^{ab}(e).$$

Therefore, one can start with the gauge fields

$$\begin{aligned} A_\mu &= i e_\mu^a \gamma_a + \frac{1}{4} \omega_\mu^{ab} \sigma_{ab}, \\ A'_\mu &= i e_\mu'^a \gamma_a + \frac{1}{4} \omega_\mu'^{ab} \sigma_{ab}, \end{aligned}$$

and introduce a pair of Higgs fields G_1 and G_2 transforming under the product representation of $SP(4) \times SP(4)$. Imposing 14 constraints on G_1 and G_2 of the form

$$Tr \left(\left(G_i \widetilde{G}_i \right)^n \right) = c_{ni}, \quad i = 1, 2,$$

breaks the symmetry spontaneously $SP(4) \times SP(4) \rightarrow SL(2, \mathbb{C})$ through a non-linear realization [15]. An action of the form

$$\int Tr \left(\alpha G_1 \widetilde{G}_2 F \wedge F + \alpha' \widetilde{G}_1 G_2 F' \wedge F' \right. \\ \left. + \beta \nabla G_1 \wedge \nabla \widetilde{G}_1 \wedge \nabla G_1 \wedge \nabla \widetilde{G}_2 + \beta' \nabla G_2 \wedge \nabla \widetilde{G}_2 \wedge \nabla G_2 \wedge \nabla \widetilde{G}_1 \right),$$

where

$$F = dA + A \wedge A \\ \widetilde{G} = C G^T C^{-1}$$

C being the charge conjugation matrix. Analysis of the quadratic part of this action reveals that one combination of e_μ^a and $e_\mu'^a$ is massless while the other combination is massive. In the unitary gauge we can choose

$$G_1 = a, \\ G_2 = b\gamma_5.$$

To illustrate how the consistency of massive gravitons is solved, we consider only one gauge group $SP(4)$ with gauge field A_μ and a Higgs multiplet G

$$G = \phi i\gamma_5 + v_a \gamma_a \gamma_5.$$

This is subject to the constraint

$$Tr (G^2) = -4a^2,$$

which in component form reads

$$\phi^2 + v_a v^a = a^2.$$

The gauge transformations

$$\delta \phi = \omega^a v_a, \\ \delta v_a = \omega_a \phi + \omega_{ab} v^b,$$

allows to choose the unitary gauge where

$$v_a = 0, \quad \phi = a.$$

An invariant action for the massive spin-2 field is [16]

$$\begin{aligned} & \int Tr (GF \wedge F + G \nabla G \nabla G \nabla G \nabla G) \\ & + \int d^4x \sqrt{g} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\nu} g^{\rho\sigma}) H_{\mu\nu}, \end{aligned}$$

where

$$H_{\mu\nu} = Tr (\nabla_\mu G \nabla_\nu G).$$

In the unitary gauge this gives the Fierz-Pauli action for a symmetric tensor $H_{\mu\nu} = h_{\mu\nu} = e_\mu^a e_{\nu a}$:

$$\int d^4x \sqrt{h} (R(h) + \Lambda) + m^2 \int d^4x \sqrt{g} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\nu} g^{\rho\sigma}) h_{\mu\nu} h_{\rho\sigma}.$$

It is also possible to analyze this action in the non-unitary gauge, where the components ϕ and v_a are kept. Because of the gauge invariance of the field e_μ^a

$$\delta e_\mu^a = \partial_\mu \omega^a + \dots$$

the dynamical degrees of e_μ^a will be identical to those of the massless graviton, thus describing helicities +2 and -2 only. The three independent components of v_a will describe the helicities +1, 0, -1

$$\int d^4x \sqrt{g} ((g^{\mu\rho} g^{\nu\sigma} - g^{\mu\nu} g^{\rho\sigma}) \partial_\mu v_\nu - m^2 v_a v^a)$$

The other helicity 0 in $h_{\mu\nu}$ is present in ϕ which couples as a scale factor

$$\int d^4x e \phi^3 (R(e) + \Lambda)$$

The ill behaved propagator can be avoided by working in the non-unitary gauge, where every helicity of the 6 degrees of freedom present in the symmetric tensor $h_{\mu\nu}$ is represented with an independent field. The discontinuity in the propagator is related to the strong coupling of the scalar longitudinal component of the graviton ϕ . One can show that the theory is well behaved below the cut-off scale [17].

2 Matrix Gravity

In D-branes, coordinates of space-time become noncommuting and $U(N)$ matrix-valued [18]

$$[X^i, X^j] \neq 0.$$

A metric on such spaces will also become matrix-valued. For example in the case of D-0 branes a matrix model action takes the form [19]

$$Tr (G_{ij} (X) \partial_0 X^i \partial_0 X^j) .$$

At very short distances coordinates of space-time can become noncommuting and represented by matrices. One may have to use the tools of noncommutative geometry of Alain Connes [3].

Developing differential geometry on such spaces is ambiguous. Defining covariant derivatives, affine connections, contracting indices, will all depend on the order these operations are performed because of noncommutativity. Some of these developments lead to inconsistencies such as the occurrence of higher spin fields [20]. In many cases studies were limited to abelian (commuting) matrices with Fierz-Pauli interactions [21]. More recently the spectral approach was taken by Avramidi [22] which implies a well defined order for geometric constructs. Experimentally [23], there is only one massless graviton. Therefore in a consistent $U(N)$ matrix-valued gravity only one massless field should result with all others corresponding to massive gravitons. The masses of the gravitons should be acquired through the Higgs mechanism.

The lesson we learned in the last section is that one should start with a large symmetry and break it spontaneously. The minimal non-trivial extension of $SL(2, \mathbb{C})$ and $U(N)$ is $SL(2N, \mathbb{C})$. This is a non-compact group. It can be taken as a gauge group only in the first order formalism, in analogy with $SL(2, \mathbb{C})$. The vierbein e_μ^a and the spin-connection ω_μ^{ab} are conjugate variables related by the zero torsion condition. The number of conditions in $T_{\mu\nu}^a = 0$ is equal to the number of independent components of ω_μ^{ab} , which can be determined completely in terms of e_μ^a . The $SL(2N, \mathbb{C})$ gauge field can be expanded in the Dirac basis in the form

$$A_\mu = ia_\mu + \gamma_5 b_\mu + \frac{i}{4} \omega_\mu^{ab} \sigma_{ab},$$

where

$$\begin{aligned} a_\mu &= a_\mu^I \lambda^I, & b_\mu &= b_\mu^I \lambda^I, & I &= 1, \dots, N^2 - 1, \\ \omega_\mu^{ab} &= \omega_\mu^{abi} \lambda^i, & i &= 0, I. \end{aligned}$$

and λ^i are the $U(N)$ Gell-Mann matrices. The analogue of $e_\mu^a \gamma_a$ is

$$L_\mu = e_\mu^a \gamma_a + f_\mu^a \gamma_5 \gamma_a,$$

where e_μ^a and f_μ^a are $U(N)$ matrices. This is equivalent to having complex matrix gravity. The zero torsion condition

$$T = dL + LA + AL = 0,$$

will give two sets of conditions

$$T_{\mu\nu}^a = 0, \quad T_{\mu\nu}^{a5} = 0,$$

which will overdetermined the variables ω_μ^{ab} .

The correct approach [24] is to consider $SL(2N, \mathbb{C}) \times SL(2N, \mathbb{C})$, or equivalently the complex extension of $SL(2N, \mathbb{C})$ as was done by Isham, Salam and Strathdee [25] for the massive spin-2 nonets. In this case

$$a_\mu = a_\mu^1 + i a_\mu^2, \quad b_\mu = b_\mu^1 + i b_\mu^2, \quad \omega_\mu^{ab} = B_\mu^{ab} + i C_\mu^{ab},$$

and the torsion zero constraints are enough to determine B_μ^{ab} and C_μ^{ab} in terms of e_μ^a , f_μ^a , a_μ and b_μ . One can write, almost uniquely, a metric independent gauge invariant action which will correspond to massless $U(N)$ gravitons

$$\int_M Tr \left(i (\alpha + \beta \gamma_5) LL' F + i (\bar{\alpha} + \bar{\beta} \gamma_5) L' L \bar{F} + (i\lambda + \gamma_5 \eta) LL' LL' \right),$$

where L' is related to L . For illustration, the form of this action in the $N = 1$ case is

$$\begin{aligned} -\frac{1}{2} \int_M d^4 x \epsilon^{\mu\nu\kappa\lambda} & \left(\left((\alpha_2 - \beta_1) e_{\mu a} e_{\nu b} + \frac{1}{2} (\alpha_1 + \beta_2) \epsilon_{abcd} e_\mu^c e_\nu^d \right) B_{\kappa\lambda}^{ab} \right. \\ & + \left((\alpha_2 + \beta_1) f_{\mu a} f_{\nu b} - \frac{1}{2} (\alpha_1 - \beta_2) \epsilon_{abcd} f_\mu^c f_\nu^d \right) C_{\kappa\lambda}^{ab} \\ & \left. + \epsilon_{abcd} \left((\lambda - \eta) e_\mu^a e_\nu^b e_\kappa^c e_\lambda^d + (\lambda + \eta) f_\mu^a f_\nu^b f_\kappa^c f_\lambda^d \right) \right). \end{aligned}$$

To give masses to the spin-2 fields, introduce the Higgs fields H and H' transforming as L and L' and constrained in such a way as to break the symmetry non-linearly from $SL(2N, \mathbb{C}) \times SL(2N, \mathbb{C})$ to $SL(2, \mathbb{C})$. We can add the mass terms

$$\int_M Tr \left((i\tau + \gamma_5 \xi) LH' LH' LL' + (i\rho + \gamma_5 \xi) HL' HL' LL' \right).$$

Some of the relevant terms in the quadratic parts of the action are, in component form [24],

$$\begin{aligned} \int d^4x \epsilon^{\mu\nu\kappa\lambda} \epsilon_{abcd} Tr \left(\alpha_1 \left\{ E_\mu^a, E_\nu'^a \right\} a_{\kappa\lambda}^2 + \alpha_2 \left\{ F_\mu^a, F_\nu'^a \right\} b_{\kappa\lambda}^2 \right. \\ \left. + \beta_1 \left\{ E_\mu^a, E_\nu'^b \right\} B_{\kappa\lambda}^{cd} + \beta_2 \left\{ F_\mu^a, F_\nu'^b \right\} C_{\kappa\lambda}^{cd} + \right. \\ \left. + \gamma_1 E_\mu^a E_\nu'^b E_\kappa^c E_\lambda'^d + \gamma_2 F_\mu^a F_\nu'^b F_\kappa^c F_\lambda'^d + \delta_1 E_\mu^a E_\nu'^b E_\kappa^c F_\lambda'^d + \delta_2 F_\mu^a F_\nu'^b F_\kappa^c E_\lambda'^d \right). \end{aligned}$$

This action is complicated because all expressions are matrix valued. Equations are solved perturbatively. The action can be determined to second order in the fields, and the spectrum found to be given by two sets of $SU(N)$ matrix-valued massive gravitons, plus two singlets of gravitons, one massless and the other is massive, as well as $SU(N) \times SU(N)$ gauge fields.

We decompose $E_{\mu a}^I$ into symmetric and antisymmetric parts

$$E_{\mu a}^I = S_{\mu a}^I + T_{\mu a}^I,$$

where $S_{\mu a}^I = S_{a\mu}^I$ is symmetric and $T_{\mu a}^I = -T_{a\mu}^I$ is antisymmetric. The symmetric part propagates while the antisymmetric part $T_{\mu\nu}^I$ couples to the Yang-Mills fields and act as auxiliary fields to give them kinetic energies. For example besides the quadratic terms for $T^{\mu\nu I}$ coming from the mass terms, we have

$$\int_M d^4x \left(\partial_\mu a_\nu^{1I} - \partial_\nu a_\mu^{1I} \right) T^{\mu\nu I},$$

as well as similar couplings to a_μ^{2I} , b_μ^{1I} , b_μ^{2I} . By eliminating the field $T_{\mu\nu}^I$ the fields a_μ^{1I} , a_μ^{2I} , b_μ^{1I} , b_μ^{2I} would acquire the regular $SU(N)$ Yang-Mills gauge field strengths. A detailed study of this system is carried in [24].

3 Conclusions

The novel features of this action are:

- The massless graviton is not introduced as a background metric but is a part of the gravitons matrix.
- Mass is generated spontaneously for the spin-2 fields.
- The massive gravitons interact in a non-trivial way.
- Correct kinetic energies for the spin-2 fields are generated, although the gauge group is noncompact. This is achieved by utilizing the first order formalism.
- Kinetic energies for the non-abelian $SU(N)$ gauge fields are generated by couplings to the antisymmetric parts of the matrix-vierbeins.
- With the requirements of gauge invariance and the restriction that the action is a four-form, the action is almost unique and is unambiguous.
- It is possible to formulate a consistent theory of matrix gravity based on $SL(2N, \mathbb{C}) \otimes SL(2N, \mathbb{C})$ gauge symmetry spontaneously broken to $SL(2, \mathbb{C})$.
- The theory unifies the massless graviton with colored massive gravitons with $SU(N) \times SU(N)$ symmetry as well as with gauge fields.

Although the gravitons are promoted to become matrix valued, the coordinates are not. Only the diagonal component of the coordinates are kept. In a more general treatment, the $4N^2$ coordinates X^μ should be used instead of the 4 coordinates x^μ used here. It is important to learn how to adopt this construction to the noncommutative geometry of Alain Connes based on spectral data and to construct Dirac operators for such spaces.

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References

- [1] J. Scherk, in *Recent Developments in Gravitation, Cargese 1978*, Editors, M. Levy and S. Deser, Plenum Press 1979.
- [2] T. Damour and I. I. Kogan, *Phys. Rev. D* **66**, 104024 (2002).
- [3] Alain Connes, *Noncommutative Geometry*, Academic Press, 1994.
- [4] A. H. Chamseddine, G. Felder and J. Fröhlich, *Comm. Math. Phys.* **155** 109 (1993).
- [5] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** 4690 (1999).
- [6] F. Lizzi, G. Mangano and G. Miele, *Mod. Phys. Lett. A* **16** 1 (2001).
- [7] M. Fierz, W. Pauli, *Proc. R. Soc. London, Ser. A* **173**, 211 (1939).
- [8] H. van Dam, M. Veltman, *Nucl. Phys. B* **22**, 397 (1970).
- [9] D. G. Boulware, S. Deser, *Phys. Rev. D* **6**, 3368 (1972).
- [10] Abdus Salam and J. Strathdee, *Phys. Rev. D* **14**, 2830 (1976).
- [11] H. Weyl, *Space, Time, Matter*, (Dover, New York, 1952).
- [12] R. Utiyama, *Phys. Rev.* **101**, 1597 (1956).
- [13] T. W. B. Kibble, *J. Math. Phys.* **2**, 212 (1961).
- [14] A. H. Chamseddine, Abdus Salam and J. Strathdee, *Nucl. Phys. B* **136**, 248 (1978).
- [15] S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177**, (1969) 2239; C. Callan, S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177**, 2247 (1969).
- [16] A. H. Chamseddine, *Phys. Lett. B* **557**, 247 (2003).
- [17] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, *Ann. Phys.* **305**, 96 (2003).
- [18] A. Connes, M. R. Douglas and A. Schwarz, *JHEP* **9802**:003 (1998).

- [19] A. H. Chamseddine, *Phys. Lett. B* **462**, 265 (1999).
- [20] S. Okubo and J. Maluf, *Phys. Rev. D* **31**, 1327 (1985).
- [21] C. Cutler and R. Wald, *Class. Quant. Grav.* **4**, 1267 (1987).
- [22] I. Avramidi, *Phys. Lett. B* **576**, 195 (2003); *Class. Quant. Grav.* **21**, 103 (2004); *Gauged Gravity via Spectral Asymptotics of Non-Laplace Type Operators*, [hep-th/0406026](#).
- [23] T. Damour, *Experimental Tests of Gravitational Theory*, in Particle Data Group, S. Eidelman et al, *Phys. Lett. B* **592** 1 (2004).
- [24] A. H. Chamseddine, *Matrix Gravity and Massive Colored Gravitons*, [hep-th/0406263](#), to be published in Physical Review D.
- [25] C. J. Isham, Abdus Salam and J. Strathdee, *Phys. Rev. D* **8**, 2600 (1973).